重庆大学
CHONGQING UNIVERSITY

Semi-analytical method for determining mode II fracture toughness and traction-separation relation directly from experimental load and displacement data

Yu Gong

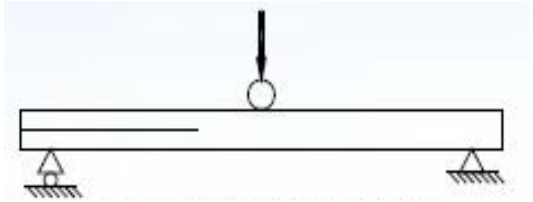
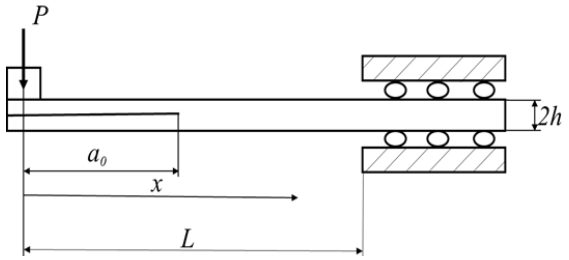
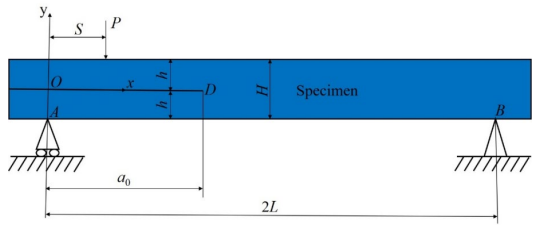
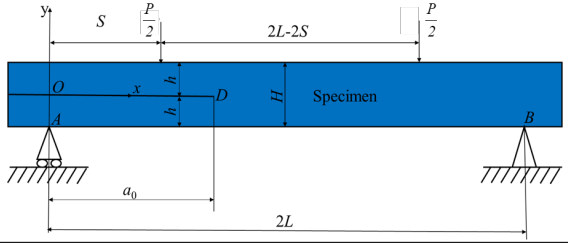
Chongqing University, China

Online Workshop

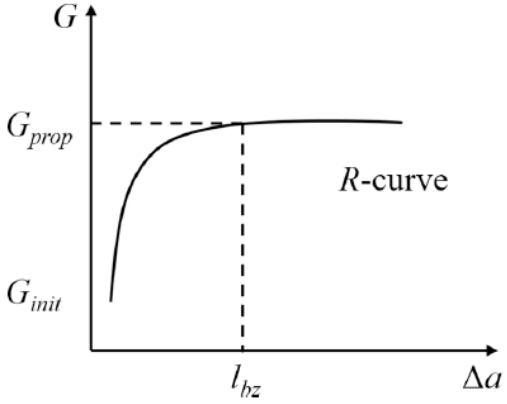
Mode II interlaminar fracture toughness and the factors affecting it

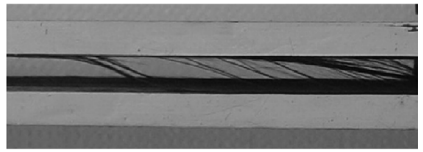
13 May 2025

Introduction-Existing test methods for mode II delamination and their characteristics

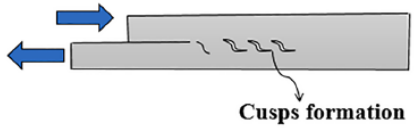
Test method	Simplified diagram	Characteristics
ENF (End-notched flexure test)		Easy to operate, but the delamination is usually unstable.
ELS (End-loaded split test)		The delamination is stable, but is prone to large displacements.
ONF (Over-notched flexure test)		The delamination is stable, but the results can be affected by friction.
4ENF (Four-point end-notched flexure test)		Easy to operate, but the results can be affected by friction.

Motivation--Why we require new data reduction method for mode II delamination?

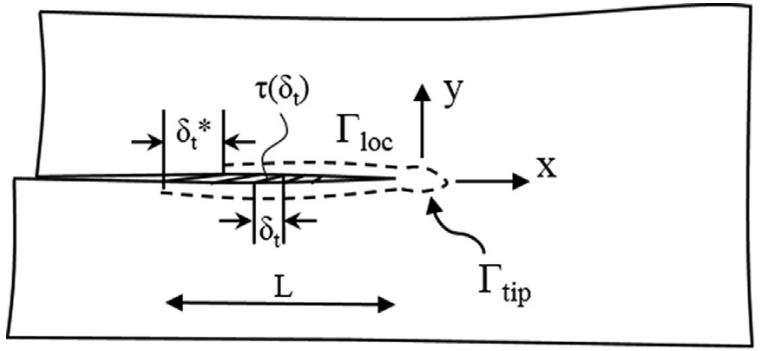




bridging



matrix plastic deformation



relative sliding displacement

Difficulty in the crack observation

- There is no distinct opening in mode II delamination, making precise determination of the crack tip location and the measurement of crack length challenging.
- It is difficult to monitor the relative sliding displacement at the pre-crack tip (if one wants to further identify the traction-separation relation).

Multiple influenced factors in mode II delamination tests

- Determination of elastic parameters E_{fx} and G_{xz}
- Friction effect in ONF and 4ENF tests
- Geometric nonlinearity in ELS tests

J Wood Sci 2000; 46(4): 273–8
Compos A Appl Sci Manuf 2007; 38(3): 785–94
J Reinf Plast Comp 2011; 30(6): 473–87.
Adv Compos Mater 2018; 27(2): 119–33.
Compos Sci Technol 2005; 65(2): 295–300.
Mech Compos Mater 2005; 41(5): 383–90.
J Compos Mater 1999; 33(1): 73–100.
Eng Fract Mech 2024; 301: 110020
Theor Appl Fract Mech 2023; 123: 103712

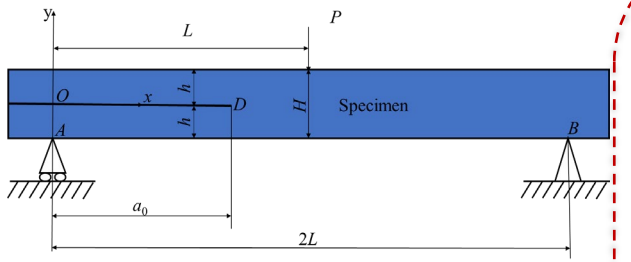
Motivation--It is difficult to balance operational convenience with simplicity in data processing

Researcher	Test type	Proposed method	Characteristics
Pérez-Galmés et al. [1]	ELS test	Closed-form solution based on the J-integral	**Operational complexity:** Additional measurement of the rotation angle is necessary
Shi and Xu [2]	ELS test	Two-dimensional finite element analysis and orthotropic scaling method	**Complexity in data processing:** The traction-separation relation cannot be determined
Xia et al. [3]	Asymmetric 4ENF test	A theoretical model based on Timoshenko beam theory	**Susceptibility to other sources of error:** The elastic properties are required to measure and traction-separation relation cannot be determined

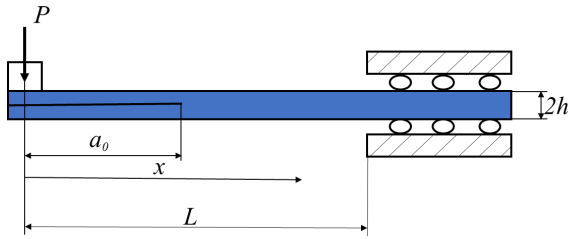
[1] M. Pérez-Galmés et al. Composites Part A: Applied Science and Manufacturing, 2016, 90: 670-7.
[2] C. Shi, W. Xu. Theoretical and Applied Fracture Mechanics, 2024, 133: 104543.
[3] K. Xia et al. Composites Part A: Applied Science and Manufacturing, 2025, 193: 108824.

Semi-analytical method for mode II delamination – an overview

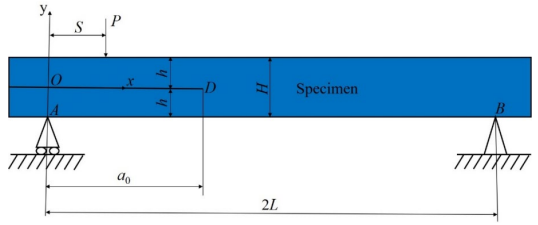
ENF



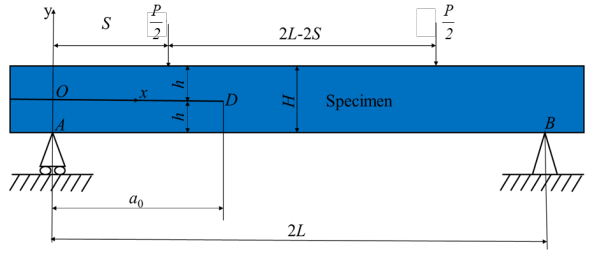
ELS



ONF



4ENF



Simplified model of the specimen → **Force analysis of the specimen** ← Friction in ONF and 4ENF tests

Establish equations between compliance *C* and crack length *a* based on Engesser-Castiglian's theorem:

$$C_{ENF}=\frac{L^3}{4E_{fx}bh^3}+\frac{3a^3}{8E_{fx}bh^3}-\frac{3\mu a^2}{4E_{fx}bh^2}+\frac{\mu^2 a}{2E_{fx}bh}+\frac{3L}{10G_{xz}bh}$$

Bring (a_0,C_0), (a_1,C_1) into equations

Solve the equations to obtain the expression of E_{fx} and G_{xz}

Bring E_{fx} and G_{xz} back into equations

Crack length and fracture toughness

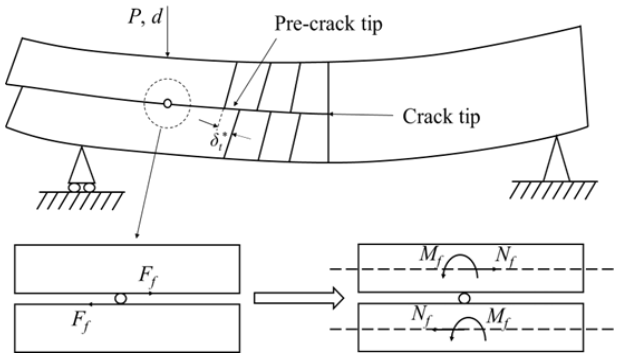
Unit load theorem

Sliding displacement at the pre-crack tip

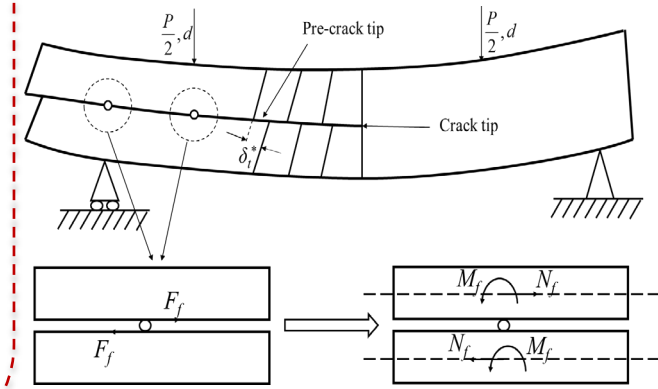
$$\tau(\delta_t^*)=\frac{\partial G_{\mathrm{II}}}{\partial \delta_t^*}$$

Traction-separation relation can be obtained based on *J*-integral method

Flowchart of semi-analytical method



Point-friction model of ONF test



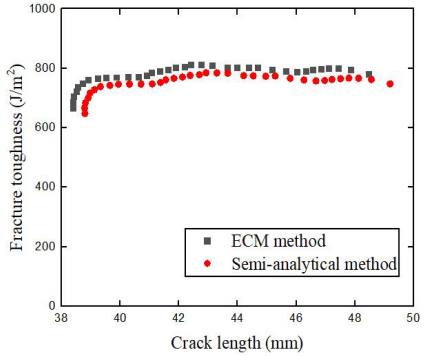
Point-friction model of 4ENF test

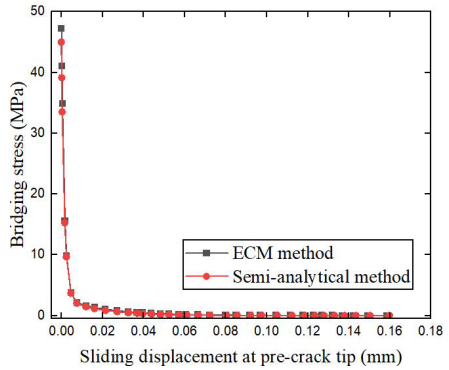
Composite laminates with various stacking sequences and material types to check the applicability of proposed semi-analytical method

Test method	Material	Stacking sequence	Material properties
ENF	E-glass/EPON828	$[0^{\circ}_{11}/45^{\circ}//0^{\circ}_{12}]$, $[0^{\circ}_{11}/60^{\circ}//0^{\circ}_{12}]$, $[0^{\circ}_{11}/90^{\circ}//0^{\circ}_{12}]$	E_{11}=35.25 GPa, E_{22}=10.82 GPa, G_{12}=4.28 GPa, v_{12}=0.27
ELS, ONF	T800 carbon/epoxy	$(45^{\circ}/-45^{\circ}/0^{\circ}_6)_S//(-45^{\circ}/45^{\circ}/0^{\circ}_6)_S$	E_{11}=195 GPa, E_{22}=8.58 GPa, G_{12}=4.57 GPa, v_{12}=0.33
	E-glass/polyester	$[0^{\circ}]_{14}$	E_{11}=33 GPa, E_{22}=E_{33}=7.2 GPa, G_{13}=3 GPa, v_{13}=0.27
4ENF	E-glass/polyester	$[45^{\circ}/-30^{\circ}/-45^{\circ}/30^{\circ}]_{4T}$	E_{11}=33 GPa, E_{22}=E_{33}=7.2 GPa, G_{13}=3 GPa, v_{13}=0.27

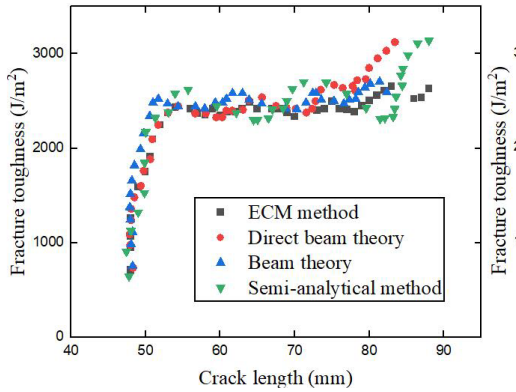
Semi-analytical method for mode II delamination – validation results

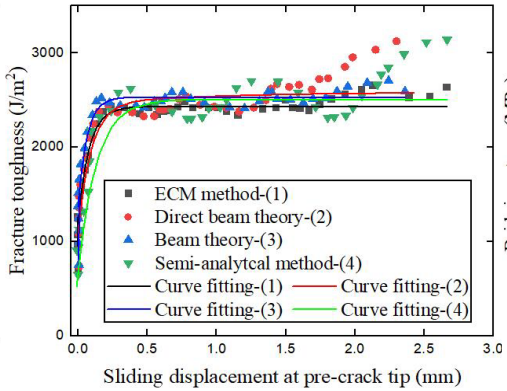
ONF test

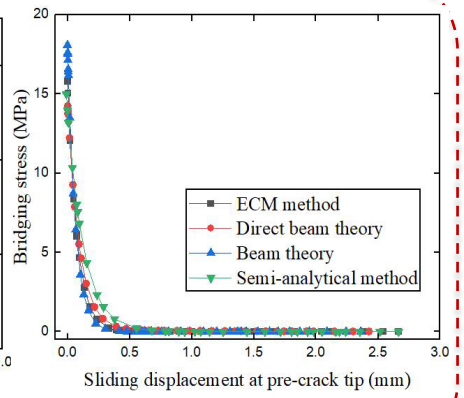




T800 carbon/epoxy laminate

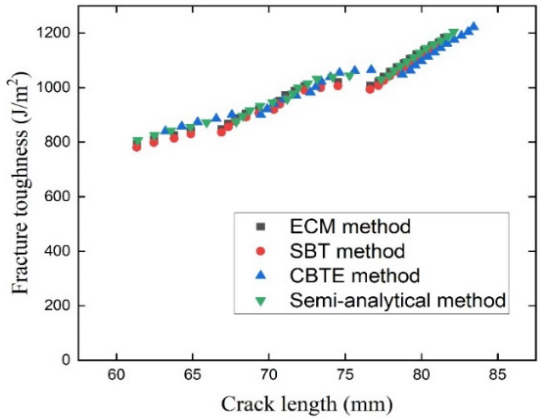


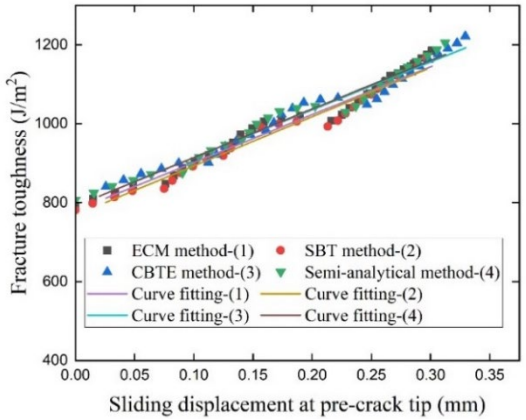




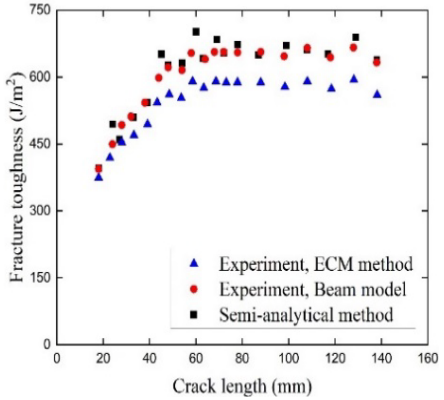
E-glass/polyester laminate

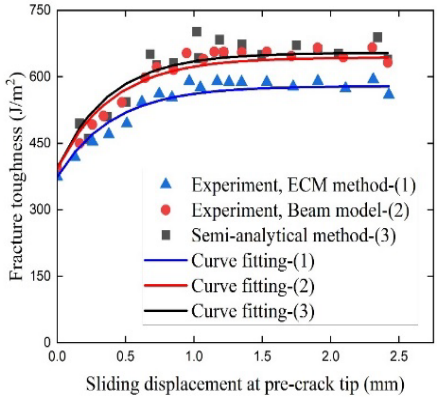
ELS test

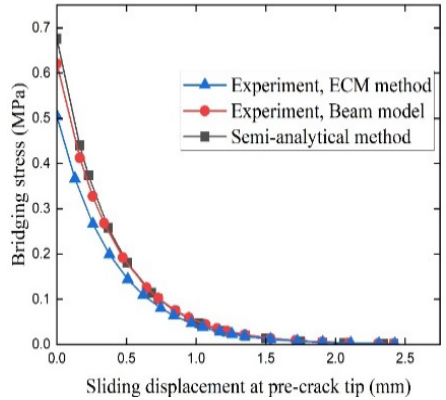




T800 carbon/epoxy laminates





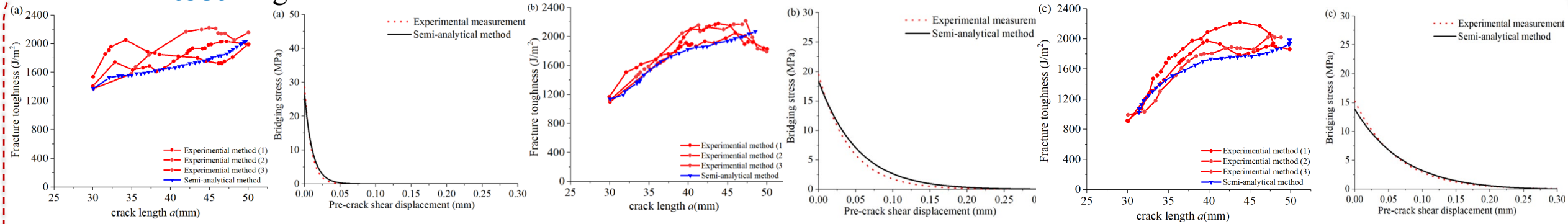


E-glass/polyester laminate

Experimental results confirm that this proposed semi-analytical method can bring satisfactory consistency in determined fracture toughness and traction-separation relation.

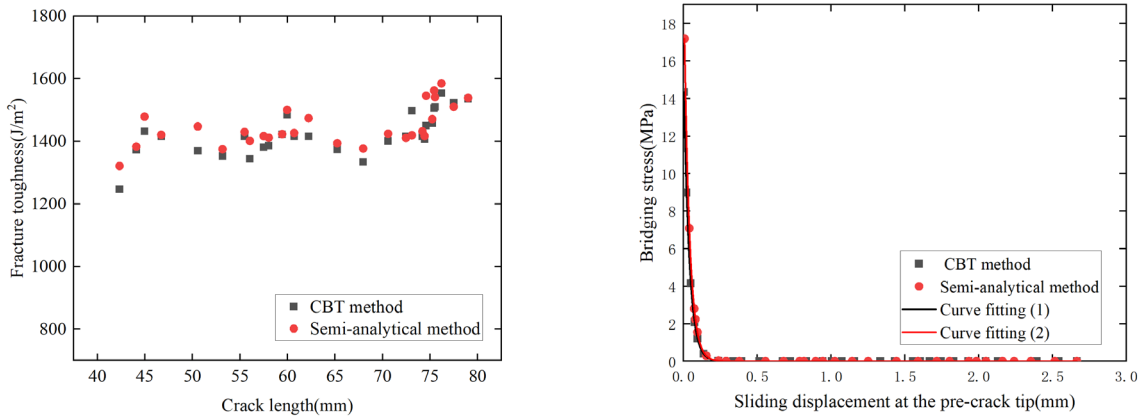
Semi-analytical method for mode II delamination – validation results

ENF test E-glass/EPON828



$[0^{o}_{11}/45^{o}//0^{o}_{12}]$ $[0^{o}_{11}/60^{o}//0^{o}_{12}]$ $[0^{o}_{11}/90^{o}//0^{o}_{12}]$

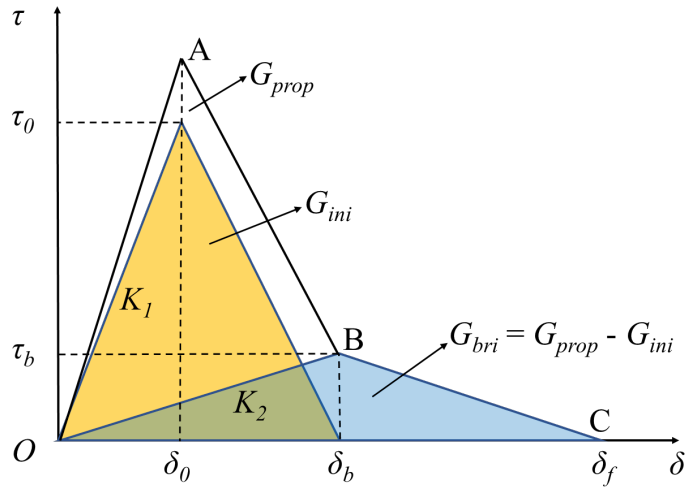
4ENF test



E-glass/polyester

Experimental results confirm that this proposed semi-analytical method can bring satisfactory consistency in determined fracture toughness and traction-separation relation.

Semi-analytical method for mode II delamination – simulation results (ONF)

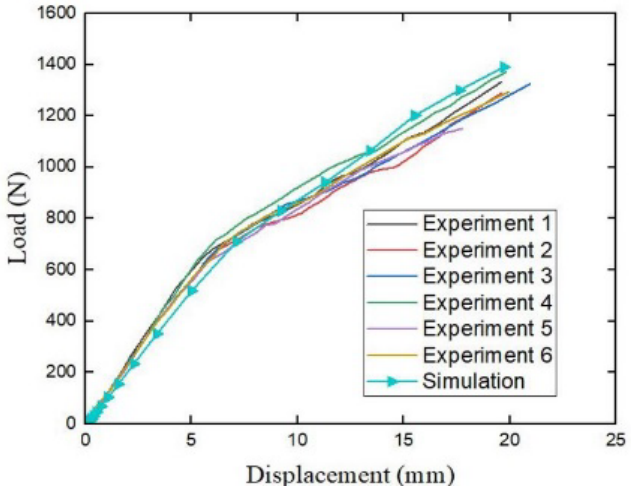


Tri-linear cohesive zone model

$$\tau = (1-D)K_0\delta$$

$$D = \begin{cases} 0 & 0 < \delta \le \delta_0 \\ (1-\frac{K_{AB}}{K_0})(1-\frac{\delta_0}{\delta}) & \delta_0 < \delta \le \delta_b \\ 1-\frac{K_{BC}}{K_0}(1-\frac{\delta_f}{\delta}) & \delta_b < \delta \le \delta_f \\ 1 & \delta > \delta_f \end{cases}$$

The comparison between experimental and numerical load-displacement responses



Material	Ultimate load (N)		Relative error (%)	Initial slope (N/mm)		Relative error (%)
	Prediction	Test		Prediction	Test	
T800/epoxy	990.4	943.7	4.9	323.3	314.2	2.9
E-glass /polyester	1390.5	1312.6	5.9	103.3	107.8	-4.1

Thanks for your attention!